

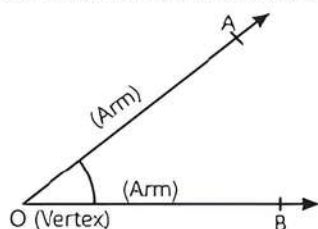
6

Lines and Angles

Fastrack Revision

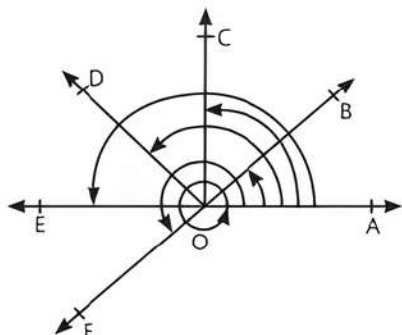
- ▶ **Line:** A geometrical object that is straight and extends indefinitely in both directions.
- ▶ **Line Segment:** A part of a line with two end points.
- ▶ **Ray:** A part of line with one end point.
- ▶ **Collinear Points:** Three or more points lying on the same line are known as collinear points. Otherwise they are non-collinear points.
- ▶ **Angle:** It is formed when two rays originate from the same end point. The rays are called arms and the end point is called vertex.

In the figure, OA and OB are arms, O is vertex and angle is AOB or BOA written as $\angle AOB$ or $\angle BOA$.



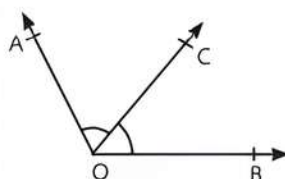
Types of Angles:

1. **Acute Angle:** An angle with measure more than 0° but less than 90° . In figure, $\angle AOB$ is acute angle.
2. **Obtuse Angle:** An angle with measure more than 90° but less than 180° . In figure, $\angle AOD$ is obtuse angle.
3. **Right Angle:** An angle with measure exactly 90° . In figure, $\angle AOC$ is right angle.
4. **Straight Angle:** An angle with measure 180° . In figure, $\angle AOE$ is straight angle.
5. **Reflex Angle:** An angle with measure more than 180° but less than 360° . In figure, $\angle AOF$ is reflex angle, when measured anticlockwise.
6. **Complete Angle:** An angle with measure 360° . In figure, $\angle AOA$ is complete angle.



Pair of Angles:

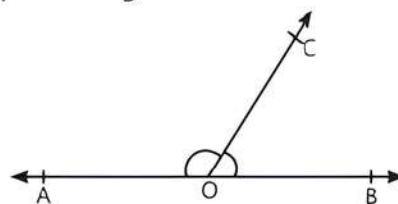
1. **Complementary Angles:** Two angles with the sum of 90° . In above figure, $\angle AOB + \angle BOC = 90^\circ$, so $\angle AOB$ and $\angle BOC$ are complementary angles.
2. **Supplementary Angles:** Two angles with the sum of 180° . In above figure, $\angle AOB + \angle BOE = 180^\circ$, so $\angle AOB$ and $\angle BOE$ are supplementary angles.
3. **Adjacent Angles:** Two angles having a common vertex and a common arm with uncommon arms on either side of the common arm. In figure, $\angle AOC$ and $\angle BOC$ are adjacent angles.



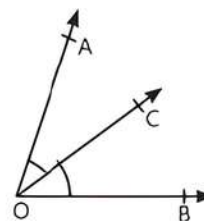
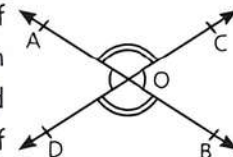
OR

When two angles are adjacent, then their sum is always equal to the angle formed by the two non-common arms. In figure, $\angle AOB = \angle AOC + \angle BOC$

4. **Linear Pair of Angles:** Two adjacent angles with the sum of 180° . In figure, $\angle AOC$ and $\angle BOC$ are linear pair of angles.

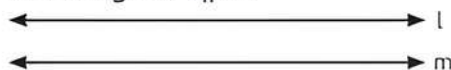


5. **Vertically Opposite Angles:** The pair of angles lying on the opposite sides of the point of intersection. In figure, ($\angle AOC$ and $\angle BOD$) and ($\angle AOD$ and $\angle BOC$) are pairs of vertically opposite angles.
6. **Bisector of an Angle:** A ray which divides an angle into two equal parts.



In figure, $\angle AOC = \angle BOC = \frac{1}{2} \angle AOB$

► **Parallel Lines:** Two lines lying in the same plane which do not intersect when produced on either side are parallel. In figure, $l \parallel m$.



► **Intersecting Lines:** Two lines which intersect at a common point are called the intersecting lines.

► **Lines Parallel to the Same Line:** Lines which are parallel to the same line are parallel to each other.

Knowledge BOOSTER

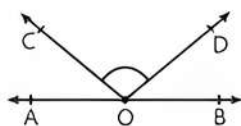
The length of perpendiculars at different points on the parallel lines is same.



Practice Exercise

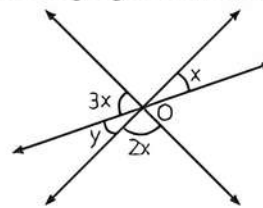
Multiple Choice Questions

- Q 1. Vertical angles that are opposite to each other, are also:
- a. not equal b. opposite
c. scalene d. equal
- Q 2. Two lines makes an angle, that point is said to be:
- a. scalene b. rays
c. segment d. vertex
- Q 3. Sum of all angles around a main point equals to:
- a. 360° b. 180°
c. 270° d. 90°
- Q 4. Angles that sum up to 90° are known as:
- a. vertical angles b. complementary angles
c. reflexive angles d. supplementary angles
- Q 5. What is the supplement of 105° ?
- a. 65° b. 75°
c. 85° d. 95°
- Q 6. What is the measure of an angle whose measure is 32° less than its supplement?
- a. 148° b. 60°
c. 74° d. 55°
- Q 7. If the supplement of an angle is 4 times of its complement, find the angle.
- a. 60° b. 50°
c. 80° d. 100°
- Q 8. If $\angle S$ and 100° form a linear pair, what is the measure of $\angle S$?
- a. 180° b. 120°
c. 90° d. 80°
- Q 9. If two complementary angles are in the 7 : 3, then angles are:
- a. $45^\circ, 45^\circ$ b. $63^\circ, 27^\circ$
c. $60^\circ, 30^\circ$ d. $50^\circ, 40^\circ$
- Q 10. In the adjoining figure, AOB is a straight line. If $\angle AOC + \angle BOD = 70^\circ$, then $\angle COD$ is equal to:



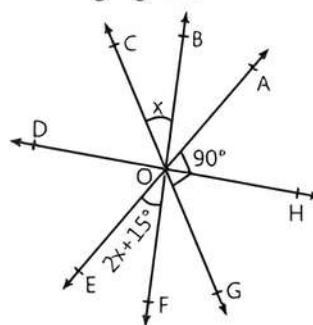
- a. 108° b. 112°
c. 110° d. 115°

Q 11. In the adjoining figure, the values of x and y are:



- a. $30^\circ, 35^\circ$ b. $30^\circ, 30^\circ$
c. $40^\circ, 40^\circ$ d. $50^\circ, 50^\circ$

Q 12. In the following figure, the value of x is:



- a. 30° b. 25°
c. 35° d. 40°

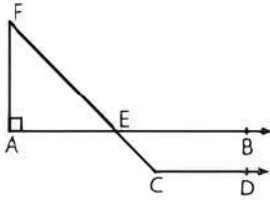
Assertion & Reason Type Questions

Directions (Q. Nos. 13-17): In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct option:

- a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.
- Q 13. Assertion (A): If angles ' x ' and ' y ' form a linear pair of angles and $x = 70^\circ$, then $y = 110^\circ$. Reason (R): Sum of linear pair of angles is always 180° .
- Q 14. Assertion (A): The pair of angles $102^\circ, 78^\circ$ is supplementary. Reason (R): The sum of two angles is 180° , then it is supplementary.



- Q 15. Assertion (A): In the adjoining figure, if $AB \parallel CD$ and $\angle E = 50^\circ$, then $\angle BEC$ is 50° .

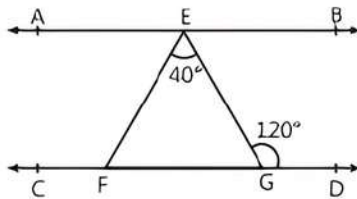


Reason (R): If sum of two angles is 90° , then it is complementary.

- Q 16. Assertion (A): The bisectors of the angles of a linear pair are at right angles.

Reason (R): If the sum of two adjacent angles is 180° , then the non-common arms of the angles are in a straight line.

- Q 17. Assertion (A): In the adjoining figure, if $AB \parallel CD$, then $\angle FGE = 60^\circ$.



Reason (R): The sum of interior angles of same side of a parallel line intersect by a transversal line is 180° .



Fill in the Blanks Type Questions

- Q 18. If the sum of two adjacent angles is, then the non-common arms of the angles form a line.
- Q 19. If the difference of two complementary angles is 40° , then angles are
- Q 20. A straight line which cuts two or more straight lines at distinct points is called a
- Q 21. If a ray stands on a line, then the sum of the two adjacent angles so formed is
- Q 22. An angle is $\frac{2}{3}$ times its supplementary angle, the measure of the angle is



True/False Type Questions

- Q 23. If one angle is five times its supplement, then angles are 30° and 144° .
- Q 24. If two lines intersect each other, then the vertically opposite angles are equal.
- Q 25. Two lines are coincident, if they have only one point common.
- Q 26. If angles forming a linear pair are equal, then each of these angles is of measure 90° .
- Q 27. The bisectors of a pair of vertically opposite angles are always at right angle.

Solutions

- (d) equal
- (d) vertex
- (a) 360°
- (b) complementary angles
- (b) Let supplement of 105° be x . Then
 $x + 105^\circ = 180^\circ \Rightarrow x = 75^\circ$
- (c) Let one of the supplement angle be x . Then other angle will be $x - 32^\circ$.



TIP

In supplementary angle, the sum of two angles is 180° .

- $$\begin{aligned} \therefore x + (x - 32^\circ) &= 180^\circ \\ \Rightarrow 2x &= 212^\circ \\ \Rightarrow x &= 106^\circ \\ \therefore \text{The required angle is } x - 32^\circ &= 106^\circ - 32^\circ = 74^\circ \end{aligned}$$
- (a) Let the angle be x . Then, its supplement is $180^\circ - x$ and its complement is $90^\circ - x$.
 According to the given condition,

$$\begin{aligned} 180^\circ - x &= 4(90^\circ - x) \\ \Rightarrow 180^\circ - x &= 360^\circ - 4x \end{aligned}$$

$$\Rightarrow 3x = 180^\circ$$

$$\Rightarrow x = 60^\circ$$

COMMON ERROR

Some of the students make mistake while consider the supplement and complement angles. So, students do more practice for such type of questions.

- (d) Given, $\angle S$ and 100° form a linear pair.
 Therefore

$$\begin{aligned} \angle S + 100^\circ &= 180^\circ \\ \Rightarrow \angle S &= 80^\circ \end{aligned}$$
- (b) Let two complementary angles be $7x$ and $3x$.
 Then,

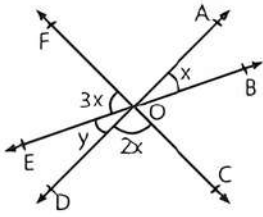
$$\begin{aligned} 7x + 3x &= 90^\circ \Rightarrow 10x = 90^\circ \\ \Rightarrow x &= 9^\circ \\ \therefore \text{The complementary angles are} & \\ 7x &= 7 \times 9 = 63^\circ \\ \text{and } 3x &= 3 \times 9 = 27^\circ \end{aligned}$$
- (c) Given, $\angle AOC + \angle BOD = 70^\circ$
 As we know that the sum of all angles of the same side of a line is 180° .

$$\therefore \angle AOC + \angle BOD + \angle COD = 180^\circ$$

$$\Rightarrow 70^\circ + \angle COD = 180^\circ$$

$$\Rightarrow \angle COD = 110^\circ$$

11. (b) $\angle EOD = \angle AOB$ [Vertically opposite angles]



$$\therefore y = x$$

Since, FC is a straight line, so sum of all angles of the same side is 180° .

$$\therefore \angle FOE + \angle EOD + \angle DOC = 180^\circ$$

$$\Rightarrow 3x + y + 2x = 180^\circ$$

$$\Rightarrow 3x + x + 2x = 180^\circ$$

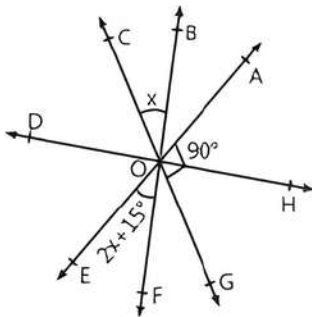
$$\Rightarrow 6x = 180^\circ \Rightarrow x = 30^\circ$$

$$\Rightarrow y = 30^\circ$$

12. (b) In the given figure,

$$\angle FOG = \angle COB$$

$$\Rightarrow \angle FOG = x \quad [\text{Vertically opposite angles}]$$



Since, AE is a straight line, so sum of all the angles of the same side of a line is 180° .

$$\therefore \angle EOF + \angle FOG + \angle GOA = 180^\circ$$

$$\Rightarrow (2x + 15^\circ) + x + 90^\circ = 180^\circ$$

$$\Rightarrow 3x + 105^\circ = 180^\circ$$

$$\Rightarrow 3x = 75^\circ \Rightarrow x = 25^\circ$$

13. (a) **Assertion (A):** Given, $x = 70^\circ$

Since, the sum of linear pair of angles is 180° .

$$\therefore x + y = 180^\circ \Rightarrow 70^\circ + y = 180^\circ$$

$$\Rightarrow y = 110^\circ$$

So, Assertion (A) is true.

Reason (R): It is true to say that the sum of linear pair of angles is 180° .

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

14. (a) **Assertion (A):** Here $102^\circ + 78^\circ = 180^\circ$.

Hence, pair of angles is a supplementary.

So, Assertion (A) is true.

Reason (R): It is true to say that the sum of two angles is 180° , then it is supplementary.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

15. (b) **Assertion (A):** Given $\angle FEA = 50^\circ$

$$\angle BEC = \angle FEA = 50^\circ$$

[Vertically opposite angles]

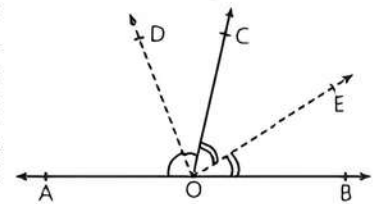
So, Assertion (A) is true.

Reason (R): It is also true to say that in complementary angles, their sum is 90° .

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

16. (b) **Assertion (A):** Let AB is a line segment and O be a point lies on the line segment.

Let OC be a ray. Construct OD and OE are the angles bisector of $\angle AOC$ and $\angle BOC$.



By definition of linear pair,

$$\angle AOC + \angle BOC = 180^\circ$$

Divide both sides by 2, we get

$$\frac{1}{2}(\angle AOC + \angle BOC) = \frac{180^\circ}{2}$$

$$\Rightarrow \frac{1}{2}\angle AOC + \frac{1}{2}\angle BOC = 90^\circ$$

$$\Rightarrow \angle DOC + \angle COE = 90^\circ$$

[\because OD and OE are the bisectors of $\angle AOC$ and $\angle BOC$,

$$\therefore \angle DOC = \frac{1}{2}\angle AOC \text{ and } \angle COE = \frac{1}{2}\angle BOC]$$

So, Assertion (A) is true.

Reason (R): It is true to say that the sum of two adjacent angles is 180° , then the non-common arms form a straight line.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

17. (b) **Assertion (A):** $\angle FGE + \angle EGD = 180^\circ$

[By linear pair]

$$\Rightarrow \angle FGE + 120^\circ = 180^\circ$$

$$\Rightarrow \angle FGE = 60^\circ$$

So, Assertion (A) is true.

Reason (R): It is true to say that the sum of interior angles of same side of a parallel line is 180° . Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

18. 180°

19. Let two complementary angles be x and $90^\circ - x$.

$$\text{Then, } (90^\circ - x) - x = 40^\circ$$

$$\Rightarrow 90^\circ - 2x = 40^\circ$$

$$\Rightarrow 2x = 50^\circ \Rightarrow x = 25^\circ$$

$$\text{and } 90^\circ - x = 90^\circ - 25^\circ = 65^\circ$$

Hence, two complementary angles are 25° and 65° .

20. transversal
 21. 180°
 22. Let required angle be x , then supplement angle will be $180^\circ - x$.

According to the given condition,

$$x = \frac{2}{3}(180^\circ - x)$$

$$\Rightarrow 3x = 360^\circ - 2x$$

$$\Rightarrow 5x = 360^\circ$$

$$\Rightarrow x = 72^\circ$$

23. False

Let one angle be x , then its supplement angle be $180^\circ - x$

According to the given condition,

$$x = 5(180^\circ - x)$$

$$\Rightarrow x = 900^\circ - 5x \Rightarrow 6x = 900^\circ$$

$$\Rightarrow x = 150^\circ$$

$$\therefore 180^\circ - x = 180^\circ - 150^\circ = 30^\circ$$

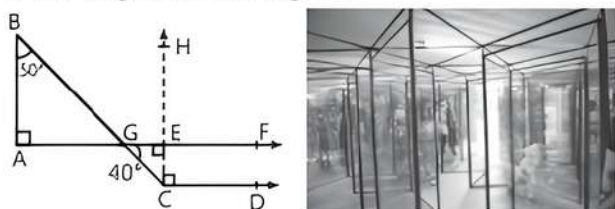
Hence, angles are 30° and 150° .

24. True
 25. False
 26. True
 27. True

Case Study Based Questions

Case Study 1

Satyam was playing with torch. He put mirrors at different places and threw torch light over them. When he threw light, it got reflected as shown below in geometrical figure.



On the basis of the above information, solve the following questions.

- Q 1. Which of the following set of points is a collinear?
 a. G, F, H b. B, A, G c. G, E, F d. E, C, D
- Q 2. The degree measure of $\angle BGA$ is:
 a. 55° b. 60° c. 50° d. 40°
- Q 3. The degree measure of $\angle BGE$ is:
 a. 140° b. 45° c. 50° d. 55°
- Q 4. The relation between line segments EF and CD is:
 a. parallel b. perpendicular
 c. intersecting d. not defined

- Q 5. In the given figure, one of the vertically opposite angle pair is:

- a. $\angle GAB$ and $\angle ABG$ b. $\angle GEH$ and $\angle HEF$
 c. $\angle AGB$ and $\angle CGE$ d. $\angle HEF$ and $\angle FEC$

Solutions

1. (c)



TIP

Three points are said to be collinear, if they lie in a line.

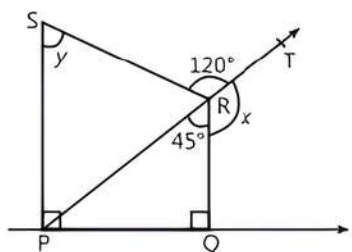
In the given options, only points G, E and F lie in a line. Hence, it is collinear.

So, option (c) is correct.

2. (d) $\angle BGA = \angle CGF$ [vertically opposite angles]
 $\therefore \angle BGA = 40^\circ$
 So, option (d) is correct.
3. (a) \therefore The sum of linear pair is 180° .
 $\therefore \angle BGA + \angle BGE = 180^\circ$
 $\Rightarrow 40^\circ + \angle BGE = 180^\circ$
 $\Rightarrow \angle BGE = 140^\circ$
 So, option (a) is correct.
4. (a) It is clear from the figure that line CH is perpendicular to the lines EF and CD. Therefore, line segments EF and CD are parallel.
 So, option (a) is correct.
5. (c) $\angle AGB$ and $\angle CGE$

Case Study 2

In a science experiment, a chemical was allowed to flow with high force and high temperature on the floor and the results were recorded. The movement of chemical is as shown in geometrical form as below.



On the basis of the above information, solve the following questions.

- Q 1. Which of the following line segment is parallel?
 a. PR and RQ b. PS and QR
 c. SR and PR d. QR and RS
- Q 2. Measure of angle x is:
 a. 130° b. 125°
 c. 135° d. 140°
- Q 3. Which of the following points are collinear?
 a. S, P and R b. P, R and Q
 c. P, R and T d. T, R and Q

Q 4. Measure $\angle SRP$ is:

- a. 50° b. 45° c. 55° d. 60°

Q 5. The sum of angles $\angle PRQ + \angle QRT$ is a

- a. complementary b. supplementary
c. Both a. and b. d. None of these

Solutions

1. (b) In the given figure, line segments SP and QR are perpendicular to the line l . Therefore, line segments PS and QR are parallel.

So, option (b) is correct.

2. (c)



TIP

A linear pair of adjacent angles is 180° .

From figure,

$$\angle PRQ + \angle TRQ = 180^\circ \quad [\text{By linear pair}]$$

$$\Rightarrow 45^\circ + x = 180^\circ$$

$$\Rightarrow x = 135^\circ$$

So, option (c) is correct.

3. (c) As we know that three points are said to be collinear, if all three points lie in a line.

In the given options, only points P, R and T lies on a line. Thus, they are collinear points.

So, option (c) is correct.

4. (d) By definition of complete angle,

$$\angle SRT + \angle TRQ + \angle PRQ + \angle SRP = 360^\circ$$

$$\therefore 120^\circ + 135^\circ + 45^\circ + \angle SRP = 360^\circ$$

$$\Rightarrow \angle SRP = 360^\circ - 300^\circ = 60^\circ$$

So, option (d) is correct.

5. (b) $\therefore \angle PRQ + \angle QRT = 45^\circ + 135^\circ = 180^\circ$

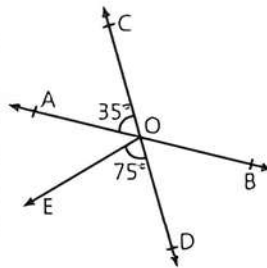
Hence, sum of two angles is a supplementary.

So, option (b) is correct.

Case Study 3

A math's teacher was teaching students about intersecting lines.

Suppose AB and CD are two intersecting lines, which meets at point O. In this point O, she draw a line OE and all these lines were making different angles with each other.



After explaining the description of the figure, she asked the following questions from the students.

On the basis of the above information, solve the following questions.

Q 1. Find the measure of $\angle BOD$.

Q 2. Check whether pair of angles $\angle AOC$ and $\angle BOC$ makes a linear pair.

Q 3. Which of the following angles form a non collinear lines?

- (i) A, O, B (ii) C, O, E

Q 4. Find the measure of $\angle AOE$.

Solutions

1. From figure,

$$\angle BOD = \angle AOC = 35^\circ$$

[Vertically opposite angles]

2. From figure, it is clear that

$$\angle AOC + \angle BOC = 180^\circ$$

[\because AB is a straight line]

Hence, $\angle AOC$ and $\angle BOC$ makes a linear pair.

3. (i) It is clear from the figure that points A, O and B form a collinear points.

(ii) It is clear from the figure that points C, O, E forms a non-collinear points.

Hence, points C, O, E form a non-collinear line.

4. From the given figure, CD is a line segment.

Therefore, the sum of all angles of the same side of a line is 180° .

$$\therefore \angle COA + \angle AOE + \angle EOD = 180^\circ$$

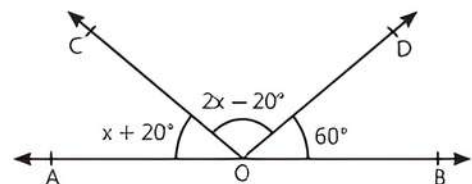
$$\Rightarrow 35^\circ + \angle AOE + 75^\circ = 180^\circ$$

$$\Rightarrow \angle AOE = 180^\circ - 110^\circ = 70^\circ$$



Very Short Answer Type Questions

Q 1. In the figure below, AOB is a straight line. Calculate the measure of $\angle COD$.



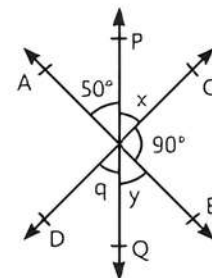
Q 2. Find the angle whose complement is equal to the angle itself.

Q 3. Two angles measure $(30^\circ - a)$ and $(125^\circ + 2a)$. If each one is the supplement of the other, then find the value of a .

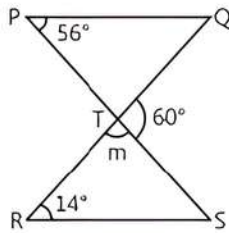
Q 4. Find the measure of an angle whose supplement is equal to angle itself.

Q 5. Two supplementary angles are in the ratio 2 : 7. Find the measure of angles.

Q 6. In given figure below, calculate the value of angle q .

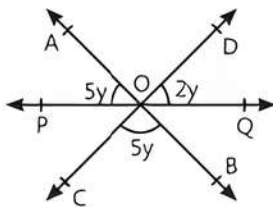


Q 7. In the given figure, if $PQ \parallel RS$, then find the measure of angle m .

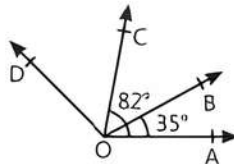


Short Answer Type-I Questions

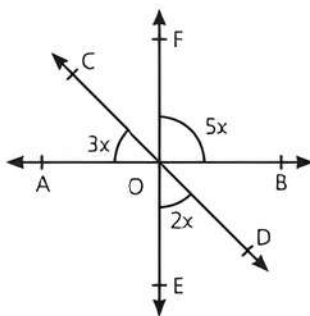
- Q 1. Find the supplement of $\frac{4}{3}$ of a right angled.
- Q 2. In the given figure, if $\angle AOP = 5y$, $\angle QOD = 2y$ and $\angle BOC = 5y$, then find the value of y .



Q 3. In the given figure, $\angle DOB = 87^\circ$ and $\angle COA = 82^\circ$. If $\angle BOA = 35^\circ$, then find $\angle COB$ and $\angle COD$.



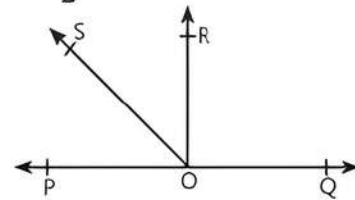
Q 4. In the given figure, lines AB, CD and EF meet at O. Find the value of x , hence find all the three indicated angles.



Short Answer Type-II Questions

- Q 1. It is given that $\angle XYZ = 64^\circ$ and XY is produced to a point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.
- Q 2. Prove that if two lines intersect each other, then the vertically opposite angles are equal.

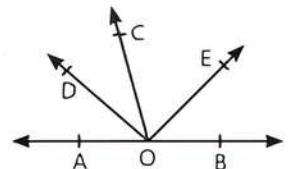
Q 3. In the given figure, POQ is a line. Ray $OR \perp PQ$. OS is another ray lying between OP and OR. Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.



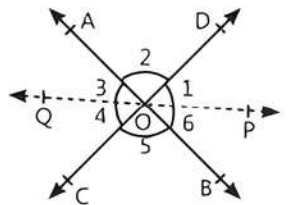
Q 4. Prove that if two lines intersect each other, then bisectors of vertically opposite angles are in the same line.

Long Answer Type Questions

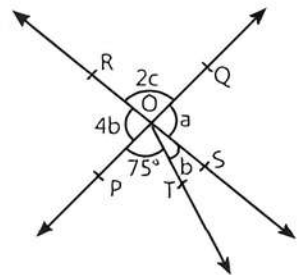
Q 1. In the given figure, OD is the bisector of $\angle AOC$, OE is the bisector of $\angle BOC$ and $OD \perp OE$. Show that the points A, O and B are collinear.



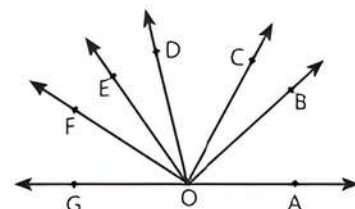
Q 2. In the given figure, AB and CD are straight lines and OP and OQ are respectively the bisectors of angles $\angle BOD$ and $\angle AOC$. Show that the rays OP and OQ are in the same line.



Q 3. In the given figure, two straight lines PQ and RS intersect each other at O. If $\angle POT = 75^\circ$, find the values of a , b and c .



- Q 4. In the given figure, $\angle AOF$ and $\angle FOG$ form a linear pair. If $\angle EOB = \angle FOC = 90^\circ$ and $\angle DOC = \angle FOG = \angle AOB = 30^\circ$.
- Find the measures of $\angle FOE$, $\angle COB$ and $\angle DOE$.
 - Name four right angles.
 - Name three pairs of adjacent complementary angles.
 - Name three pairs of adjacent angles.



Solutions

Very Short Answer Type Questions

- \therefore Straight line makes an angle of 180° .

$\therefore x + 20^\circ + 2x - 20^\circ + 60^\circ = 180^\circ$

$\Rightarrow 3x = 180^\circ - 60^\circ = 120^\circ$

$\Rightarrow x = 40^\circ$

Hence, $\angle COD = 2x - 20^\circ = 2 \times 40 - 20^\circ = 60^\circ$
- Let the measure of an angle be x , then measure of its complement is also x .



TiP

In a complementary angles, the sum of two angles is 90° .

- \therefore Sum of the measures of complementary angles = 90°
- $\therefore x + x = 90^\circ$
- $\Rightarrow 2x = 90^\circ \Rightarrow x = 45^\circ$
- Hence, the required angle is 45° .

3.



TiP

In a supplementary angles, the sum of two angles is 180° .

- Angles $(30^\circ - a)$ and $(125^\circ + 2a)$ are supplementary to each other.
- $\therefore 30^\circ - a + 125^\circ + 2a = 180^\circ$
- $\Rightarrow a = 180^\circ - 155^\circ = 25^\circ$
- Hence, the value of a is 25° .

- Let the measure of an angle be x , then measure of its supplement is also x .

- \therefore Sum of supplementary angles = 180° .
- $\therefore x + x = 180^\circ$
- $\Rightarrow 2x = 180^\circ$
- $\Rightarrow x = 90^\circ$

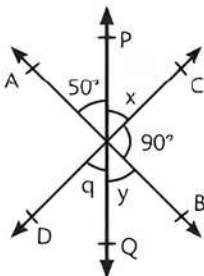
Hence, the required angle is 90° .

- Let the two angles be $2x$ and $7x$.

- $\therefore 2x + 7x = 180^\circ$
- $\Rightarrow 9x = 180^\circ$
- $\Rightarrow x = 20^\circ$
- Hence, the angles are $2x = 2 \times 20^\circ = 40^\circ$ and $7x = 7 \times 20^\circ = 140^\circ$

- From figure, $x + 50^\circ + 90^\circ = 180^\circ$

[Straight angle]



$\Rightarrow x = 40^\circ$
or $q = x = 40^\circ$

[Vertically opposite angles]

Hence, the value of angle q is 40° .

- \therefore RQ is a straight line.

$\therefore m + 60^\circ = 180^\circ$
 $\Rightarrow m = 120^\circ$

Short Answer Type-I Questions

- $\frac{4}{3}$ of a right angled = $\frac{4}{3} \times 90^\circ = 120^\circ$

\therefore Sum of supplementary angles is 180° .

\therefore Supplement of $120^\circ = 180^\circ - 120^\circ = 60^\circ$

Hence, the required supplement is 60° .
- Given, $\angle QOD = 2y$ and $\angle AOP = 5y$

$\therefore \angle POC = \angle QOD = 2y$
and $\angle QOB = \angle AOP = 5y$

[Vertically opposite angles]

From figure,

$5y + 2y + 5y = 180^\circ$

[\therefore The sum of all angles of the same side of a line is 180°]

$\Rightarrow 12y = 180^\circ \Rightarrow y = 15^\circ$

Hence, the value of y is 15° .

- Given, $\angle COA = 82^\circ$

$\Rightarrow \angle COB + \angle BOA = 82^\circ$

$\Rightarrow \angle COB + 35^\circ = 82^\circ$ [$\therefore \angle BOA = 87^\circ$]

$\Rightarrow \angle COB = 82^\circ - 35^\circ = 47^\circ$

Similarly, $\angle DOB = \angle COD + 47^\circ$

$\Rightarrow 87^\circ = \angle COD + 47^\circ$ [$\therefore \angle DOB = 87^\circ$]

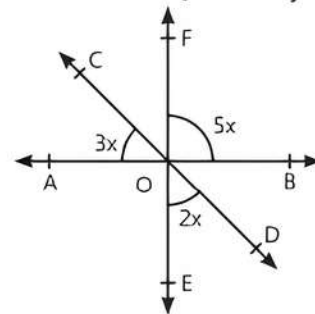
$\Rightarrow \angle COD = 87^\circ - 47^\circ = 40^\circ$

Hence, $\angle COB = 47^\circ$ and $\angle COD = 40^\circ$.

- From figure,

$\angle COF = \angle DOE = 2x$

[Vertically opposite angles]



Since, AB is a straight line, so sum of all angles on the same side of a line is 180° .

$\therefore 3x + 2x + 5x = 180^\circ$

$\Rightarrow 10x = 180^\circ$

$\Rightarrow x = 18^\circ$

$\therefore \angle AOC = 3x = 3 \times 18^\circ = 54^\circ$

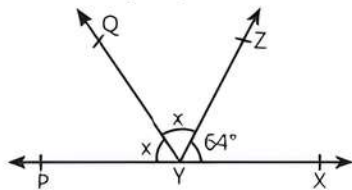
$\angle BOF = 5x = 5 \times 18^\circ = 90^\circ$

and $\angle DOE = 2x = 2 \times 18^\circ = 36^\circ$



Short Answer Type-II Questions

1. It is given that ray YQ bisects $\angle ZYP$. So let



$$\angle PYQ = \angle QYZ = x$$

$$\Rightarrow \angle PYZ = 2x$$

Since, QY stands on ray PX.

$$\angle PYZ + \angle ZYX = 180^\circ \quad [\text{By linear pair axiom}]$$

$$\Rightarrow 2x + 64^\circ = 180^\circ \quad [\because \angle ZYX = \angle XYZ = 64^\circ]$$

$$\Rightarrow 2x = 180^\circ - 64^\circ$$

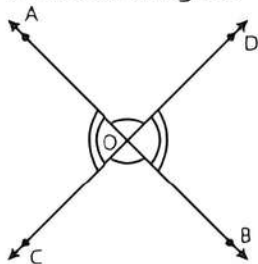
$$\Rightarrow 2x = 116^\circ$$

$$\Rightarrow x = 58^\circ$$

$$\therefore \angle XYQ = \angle XYZ + \angle ZYQ = 64^\circ + 58^\circ = 122^\circ$$

$$\text{and, reflex } \angle QYP = 360^\circ - \angle QYP = 360^\circ - x \\ = 360^\circ - 58^\circ = 302^\circ$$

2. Let the two lines AB and CD intersect each other at point O as shown in figure:



Since, ray OA stands on line CD at O.

$$\therefore \angle AOC + \angle AOD = 180^\circ \quad \dots(1)$$

Also, ray OC stands on line AB at O.

$$\therefore \angle AOC + \angle COB = 180^\circ \quad \dots(2)$$

From eqs. (1) and (2), we get

$$\angle AOC + \angle AOD = \angle AOC + \angle COB$$

$$\Rightarrow \angle AOD = \angle COB$$

Similarly, we can prove

$$\angle AOC = \angle BOD \quad \text{Hence proved}$$

3. **Given:** $OR \perp PQ$

$$\text{To Prove: } \angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$

$$\text{Proof: } \because OR \perp PQ$$

$$\therefore \angle POR = \angle ROQ = 90^\circ$$

$$\therefore \angle POS + \angle ROS = 90^\circ$$

$$\Rightarrow \angle ROS = 90^\circ - \angle POS$$

Adding $\angle ROS$ on both sides, we get

$$2\angle ROS = 90^\circ - \angle POS + \angle ROS$$

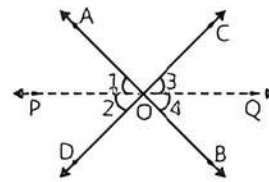
$$\Rightarrow 2\angle ROS = (90^\circ + \angle ROS) - \angle POS$$

$$= \angle QOS - \angle POS$$

$$[\because 90^\circ + \angle ROS = \angle QOS]$$

$$\therefore \angle ROS = \frac{1}{2}(\angle QOS - \angle POS) \quad \text{Hence proved}$$

4. Let AB and CD be two lines intersecting each other, OP and OQ are bisectors of $\angle AOD$ and $\angle BOC$.



$$\text{From figure, } \angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4 \quad \dots(1)$$

$$\angle AOC = \angle BOD$$

[Vertically opposite angles] $\dots(2)$

Adding eqs. (1) and (2), we get

$$\angle 1 + \angle AOC + \angle 3 = \angle 2 + \angle BOD + \angle 4 \quad \dots(3)$$

At point O, complete angle is

$$\angle 1 + \angle AOC + \angle 3 + \angle 2 + \angle BOD + \angle 4 = 360^\circ$$

$$\Rightarrow \angle 1 + \angle AOC + \angle 3 + \angle 1 + \angle AOC + \angle 3 = 360^\circ \quad [\text{From eq. (3)}]$$

$$\Rightarrow 2(\angle 1 + \angle AOC + \angle 3) = 360^\circ$$

$$\Rightarrow \angle 1 + \angle AOC + \angle 3 = 180^\circ$$

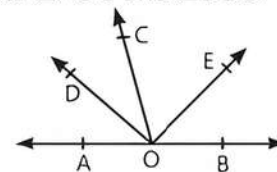
$$\text{and } \angle 2 + \angle BOD + \angle 4 = 180^\circ$$

Hence, OP and OQ are in the same line.

Hence proved

Long Answer Type Questions

1. **Given:** In figure, $OD \perp OE$, OD and OE are the bisectors of $\angle AOC$ and $\angle BOC$.



To Show: Points A, O and B are collinear i.e., AOB is a straight line.

Proof: Since, OD and OE bisect angles $\angle AOC$ and $\angle BOC$, respectively.

$$\therefore \angle AOC = 2\angle DOC \quad \dots(1)$$

$$\text{and } \angle COB = 2\angle COE \quad \dots(2)$$

On adding eqs. (1) and (2), we get

$$\angle AOC + \angle COB = 2\angle DOC + 2\angle COE$$

$$\Rightarrow \angle AOC + \angle COB = 2(\angle DOC + \angle COE)$$

$$\Rightarrow \angle AOC + \angle COB = 2\angle DOE$$

$$\Rightarrow \angle AOC + \angle COB = 2 \times 90^\circ \quad [\because OD \perp DE]$$

$$\Rightarrow \angle AOC + \angle COB = 180^\circ$$

$$\therefore \angle AOB = 180^\circ$$

So, $\angle AOC$ and $\angle COB$ are forming linear pair.

Also, AOB is a straight line.

Hence, points A, O and B are collinear.

2. In order to prove that OP and OQ are in the same line, it is sufficient to prove that $\angle POQ = 180^\circ$.

Now, OP is the bisector of $\angle BOD$

$$\Rightarrow \angle 1 = \angle 6 \quad \dots(1)$$

And, OQ is the bisector of $\angle AOC$.

$$\therefore \angle 3 = \angle 4 \quad \dots(2)$$

Clearly, $\angle 2$ and $\angle 5$ are vertically opposite angles.

$$\therefore \angle 2 = \angle 5 \quad \dots(3)$$

Since, the sum of the angles formed at a point is 360° .

$$\begin{aligned} \therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 &= 360^\circ \\ \Rightarrow (\angle 1 + \angle 6) + (\angle 3 + \angle 4) + (\angle 2 + \angle 5) &= 360^\circ \\ \Rightarrow (\angle 1 + \angle 1) + (\angle 3 + \angle 3) + (\angle 2 + \angle 2) &= 360^\circ \\ &\text{[Using eqs. (1), (2) and (3)]} \\ \Rightarrow 2(\angle 1 + \angle 3 + \angle 2) &= 360^\circ \\ \Rightarrow \angle 1 + \angle 3 + \angle 2 &= 180^\circ \\ \Rightarrow \angle POQ &= 180^\circ \end{aligned}$$

Hence, rays OP and OQ are in the same straight line.

3. Since, OR and OS are in the same line. Therefore, the sum of all angles of the same side of a line is 180° .

$$\begin{aligned} \text{i.e., } \angle ROP + \angle POT + \angle TOS &= 180^\circ \\ \Rightarrow 4b + 75^\circ + b &= 180^\circ \\ \Rightarrow 5b + 75^\circ &= 180^\circ \\ \Rightarrow 5b &= 105^\circ \Rightarrow b = 21^\circ \end{aligned}$$

Since, PQ and RS intersect at O.

$$\begin{aligned} \therefore \angle QOS &= \angle POR \\ &\text{[vertically opposite angles are equal]} \\ \Rightarrow a &= 4b \quad [\because b = 21^\circ] \\ \Rightarrow a &= 4 \times 21^\circ = 84^\circ \end{aligned}$$

Now, OR and OS are in the same line.

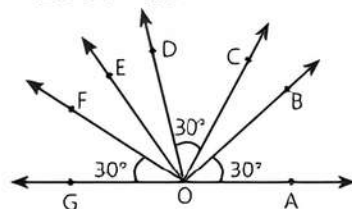
$$\begin{aligned} \therefore \angle ROQ + \angle QOS &= 180^\circ \quad \text{[by linear pair]} \\ \Rightarrow 2c + a &= 180^\circ \\ \Rightarrow 2c + 84^\circ &= 180^\circ \quad [\because a = 84^\circ] \\ \Rightarrow 2c &= 96^\circ \Rightarrow c = 48^\circ \end{aligned}$$

Hence, $a = 84^\circ$, $b = 21^\circ$ and $c = 48^\circ$.

4. Given, $\angle DOC = \angle FOG = \angle AOB = 30^\circ$

Let $\angle FOE = x$, $\angle COB = y$

and $\angle DOE = z^\circ$.



Since, AG is a straight line.

$$\begin{aligned} \therefore \angle FOG + \angle FOE + \angle DOE + \angle DOC \\ + \angle COB + \angle AOB &= 180^\circ \\ \Rightarrow 30 + x + z + 30 + y + 30^\circ &= 180^\circ \\ \Rightarrow x + y + z &= 90^\circ \quad \dots(1) \\ \text{Also, } \angle FOC &= 90^\circ \\ \Rightarrow \angle FOE + \angle DOE + \angle DOC &= 90^\circ \\ \Rightarrow x + z + 30^\circ &= 90^\circ \\ \Rightarrow x + z &= 60^\circ \quad \dots(2) \\ \text{and } \angle EOB &= 90^\circ \\ \Rightarrow \angle DOE + \angle DOC + \angle COB &= 90^\circ \\ \Rightarrow z + 30^\circ + y &= 90^\circ \\ \Rightarrow z + y &= 60^\circ \quad \dots(3) \end{aligned}$$

Solving eqs. (1), (2) and (3), we get

$$x = y = z = 30^\circ$$

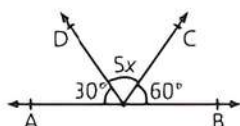
- (i) $\angle FOE = \angle COB = \angle DOE = 30^\circ$
 (ii) $\angle AOD$, $\angle BOE$, $\angle COF$, $\angle DOG$
 (iii) $\angle AOB$, $\angle BOD$; $\angle AOC$, $\angle COD$; $\angle BOC$, $\angle COE$
 (iv) $\angle BOC$, $\angle COD$; $\angle COD$, $\angle DOE$; $\angle DOE$, $\angle EOF$



Chapter Test

Multiple Choice Questions

- Q 1. In the given figure, AOB is a straight line. The value of x is:



- a. 27° b. 20° c. 18° d. 29°

- Q 2. Two complementary angles are such that twice the measure of one is equal to three times the measure of the other. The larger of the two measures is:

- a. 54° b. 62°
 c. 36° d. 70°

Assertion and Reason Type Questions

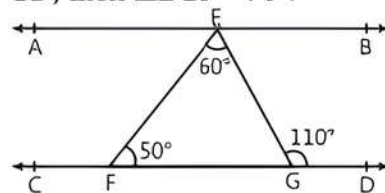
Directions (Q. Nos. 3-4): In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct option:

- a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
 b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
 c. Assertion (A) is true but Reason (R) is false.
 d. Assertion (A) is false but Reason (R) is true.

- Q 3. Assertion (A): The pair of angles 110° and 70° is supplementary.

Reason (R): The sum of two angles is not 180° , then it is supplementary.

- Q 4. Assertion (A): In the adjoining figure, if $AB \parallel CD$, then $\angle EGF = 70^\circ$.



Reason (R): The linear pair of angles is supplementary.

Fill in the Blanks

- Q 5. The supplement of an acute angle is
- Q 6. If two lines are intersecting then vertically opposite angles are

True/False

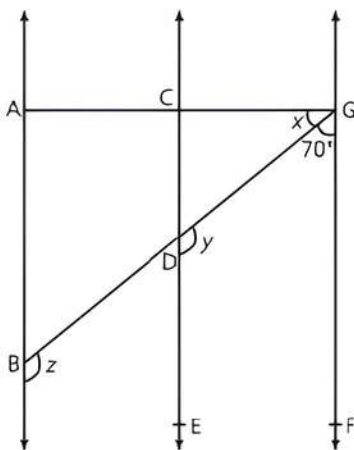
- Q 7. Three lines are concurrent if they intersect at a common point.
- Q 8. Two adjacent angles are said to form a linear pair of angles, if their non-common arms are two opposite rays.

Case Study Based Question

- Q 9. Suryansh recently bought a smart watch from a shop. It was having a lot of features and he was keen to test those feature and one of the feature was tracking the distance covered and was showing it on a digital map.



The geometrical map is shown below:
In this figure $AB \parallel CD \parallel GF$ and $AG \perp AB$.



On the basis of the given information, solve the following questions.

- (i) Find the measure of angle x .
- (ii) Find the value of y .

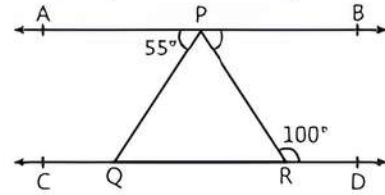
OR

In the given figure, if $z = y$, then write the value of $x + y + z$.

- (iii) Find the measure of $\angle BDC$.

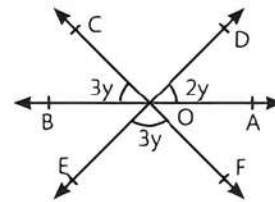
Very Short Answer Type Questions

- Q 10. Find the measure of an angle which is complement of itself.
- Q 11. In the given figure, $AB \parallel CD$. If $\angle APQ = 55^\circ$ and $\angle PRD = 100^\circ$, then find $\angle QPR$.



Short Answer Type-I Questions

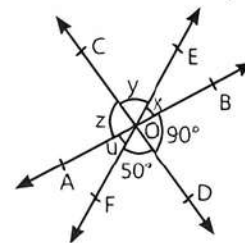
- Q 12. In the given figure, determine the value of y .



- Q 13. Prove that the bisector of a pair of vertically opposite angles are in the same straight line.

Short Answer Type-II Questions

- Q 14. In the given figure, three coplanar lines intersect at a point O, forming angles as shown in the figure. Find the values of x , y , z and u .



- Q 15. Prove that two lines that are respectively perpendicular to two intersecting lines intersect each other.

Long Answer Type Question

- Q 16. In the given figure, two straight lines PQ and RS intersect each other at O. If $\angle POT = 80^\circ$, then find the values of a , b and c .

